**Monte Carlo Methods and Applications**

**3 course, specialty – Mathematics**

# Cluster A

1. Random variables. Characteristics. Distribution function. Probability Density Function (PDF).
2. Main idea of solution of the problem by Monte Carlo methods.
3. Expectation and its property’s.
4. Chebyshev inequality.
5. Variance and its property’s.
6. Random Variable Generation (RVG). Uniformly distributed random variable in interval . Algorithms.
7. Modeling some discrete integer random variables. Recurrent formula of modeling.
8. Definite integral computing. Variance estimate. Algorithms.
9. Direct modeling (standard method of modeling).
10. Essential sampling method. Theorem.
11. Modeling isotropic vector in three-dimensional space. Algorithms.
12. Discrete Markov chain. Modeling of discrete Markov chains.
13. Solution of the linear algebraic equations system (LAES). Algorithms.
14. Homogeneous Markov chain. Initial density, density of probabilities of conversion , conversion density  and probability of break .
15. Second-order integral equations. Neumann series and its convergence.
16. Second-order conjugate integral equations. Conjugate integral operators, spaces.
17. Modeling homogeneous Markov chains. Algorithm.

**Cluster B**

1. Solution of the conjugate LAES.
2. Homogeneous Markov chains which breaking with probability 1. Computing of the probabilities of events: ,  и . Breaking condition of chains after finite number conversion with probability 1.
3. Solution of the conjugate integral equation.
4. Finiteness of expectation . Sufficient condition of finiteness .
5. Basic estimate of functionals .
6. Theorem on the unbiased of basic estimate . Proof of the theorem.
7. Some estimates of the integral equations solution.
8. Variance of the basic estimates .
9. Estimate on absorptions .
10. Estimate with zero variance. “Ideal” Markov chain.
11. Example of solution of integral equation by Monte Carlo methods.
12. Queuing System (QS) calculus. Scheme of calculus. Algorithm.
13. Calculus of passing the neutrons through plate. Statement of the problem. Scheme of calculus the real trajectories by modeling.
14. “Random walk on spheres” process. Definition and properties of “Random walk on spheres”.
15. Construction and ground of the algorithm “Random walk on spheres” for solution Dirichlet problem for Helmholtz equations.
16. Low estimate of probability of breaks of the “random walk on spheres” process.
17. Modeling algorithm of homogeneous Markov chains and theirs relation with integral

equations.

**Cluster C**

1. Construction and ground of the algorithm “Random walk on spheres” for solution Dirichlet problem for Helmholtz equations. Unbiased estimate of solution, but a unreliazable estimate.
2. Construction and ground of the algorithm “Random walk on spheres” for solution Dirichlet problem for Helmholtz equations. biased estimate of solution, but a realizable estimate.
3. Construction and ground of the algorithm “Random walk on spheres” for solution Dirichlet problem for Helmholtz equations. The right-hand member of integral equation on one random elements estimate. (All integrals estimate by means at one density).
4. Construction and ground of the algorithm “Random walk on spheres” for solution Dirichlet problem for Helmholtz equations. The algorithm of Monte Carlo methods for estimate of solution at the given point .
5. Construction and ground of the algorithm “Random walk on spheres” for solution Dirichlet problem for Poisson equations.
6. Construction and ground of the algorithm “Random walk on spheres” for solution Dirichlet problem for Poisson equations. Unbiased estimate of solution, but a unreliazable estimate.
7. Construction and ground of the algorithm “Random walk on spheres” for solution Dirichlet problem for Poisson equations. biased estimate of solution, but a realizable estimate.
8. Construction and ground of the algorithm “Random walk on spheres” for solution Dirichlet problem for Poisson equations. The right-hand member of integral equation on one random elements estimate. (All integrals estimate by means at one density).
9. Construction and ground of the algorithm “Random walk on spheres” for solution Dirichlet problem for Poisson equations. The algorithm of Monte Carlo methods for estimate of solution at the given point .
10. Green function of operator  for balls of radius  with center at the point .
11. Green function of Laplace operator for balls of radius  with center at the point .
12. Estimate of the norm of integral operator. Convergence of Neumann series.
13. Theorem on finiteness’s of estimate of variance.
14. Estimate of derivatives on the solution of Dirichlet problem for Poisson equations by Monte Carlo methods.
15. Estimate of derivative to parameter on solution of Dirichlet problem for Helmholtz equation at the given point 
16. Estimate of the integral in expression on derivative calculation from solution one by one random “node”. General density for estimate of integrals in expression on derivative calculation at the points  from solution one by one random “node”.
17. Estimate of the integrals under the expectation sign one by one random “node” (distribution density).

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